

INMO Practice Test-I
Maharashtra and Goa Region

Time: 4 Hours

[100 marks]

1. The real numbers a, b, c, d satisfy simultaneously the equations

$$abc - d = 1, bcd - a = 2, cda - b = 3, dab - c = -6.$$

Prove that $a + b + c + d \neq 0$. [16]

2. Consider a triangle ABC and let M be the midpoint of the side BC . Suppose $\angle MAC = \angle ABC$ and $\angle BAM = 105^\circ$. Find the measure of $\angle ABC$. [16]

3. There are two piles of coins, each containing 2010 pieces. Two players A and B play a game taking turns (A plays first). At each turn, the player on play has to take one or more coins from one pile or exactly one coin from each pile. Whoever takes the last coin is the winner. Which player will win if they both play in the best possible way? [17]

4. Find all positive integers n such that $36^n - 6$ is a product of two or more consecutive positive integers. [17]

5. Consider a triangle ABC with $\angle ACB = 90^\circ$. Let F be the foot of the altitude from C . Circle w touches the line segment FB at point P , the altitude CF at point Q and the circumcircle of ABC at point R . Prove that points Q, R are collinear and $AP = AC$. [17]

6. The real positive numbers x, y, z satisfy the relations

$$x \leq 2, y \leq 3, x + y + z = 11.$$

Prove that $\sqrt{xyz} \leq 6$. [17]

Indian National Mathematics Olympiad Examination 2011 will be held on 6th February 2011. The students from Nagpur, Amravati, Akola and Wardha will write the examination at Hislop College, Nagpur while all other students from Maharashtra and Goa region selected to write INMO will write the examination at Fergusson College, Pune. The examination will be held from 1.00 pm to 5.00 p.m. All the selected students should report at their respective centers not later than 12.30p.m. on the day of examination. They should bring along with them their school/college identity card and writing material. Use of calculators is not allowed.