Institute of Mathematical Sciences, Chennai 600113.

ATM Workshop on Toplogy of Manifolds

01–13 February, 2009.

Syllabus

The Workshop is devoted to understanding topology of manifolds. There are three main topics which will be focussed on during the Workshop. The syllabus for each topic is listed below.

Topics of lectures 1. Aspherical manifolds, (Goutam Mukherjee)
2. Structures on manifolds, (P. Sankaran)
3. Quasi-toric manifolds (Mainak Poddar).

1. *Aspherical manifolds* Aspherical manifolds are closed manifolds M whose universal coverings are contractible. Thus they are $K(\pi, 1)$ spaces, where $\pi = \pi(M)$. Aspherical manifolds occur naturally in several branches in Mathematics. The following are some examples: 1. In several complex variables, Riemann surfaces, quotients of bounded domains, hyperbolic manifolds, holomorphic Seifert fiberings. 2. In differential geometry: manifolds whose sectional curvatures are less or equal to zero. 3. In Lie group theory: Double coset spaces $KG\Gamma$, where G is connected, K maximal compact and Γ a torsion free uniform lattice.

Which groups appear as the fundamental groups of aspherical manifolds? In this course, we shall discuss a method of constructing model aspherical manifolds M with fundamental group for some specific groups . We also discuss the relationship between the group of self-homotopy equivalence of M to the group of homeomorphisms of M. In particular, this helps us to determine the compact Lie subgroups of the group of homeomorphisms of M. Finally we will indicate how a self homotopy equivalence can be deformed to a homeomorphisms.

References

1. P.E. Conner, F. Raymonds, Proc. second conference of compact transformation groups, Lecture Notes in Math.Vol 299,1972. pp.1-75.
2. P.E. Conner, F. Raymonds, Realizing finite groups of homeomorphism from homotopy classes of self-homotopy equivalences, manifolds-Tokyo 1973, Univ. of Tokyo press, 1975, pp. 231-238.
3. P.E. Conner, F. Raymonds, Actions of compact Lie groups on aspherical manifolds, Topology of manifolds, (Proc. Inst. Univ. Georgia, 1969), Markham,

2. *Quasi-toric manifolds* The quasi-toric manifolds are topological analogues of smooth projective toric varieties. They were introduced by Davis and Januszkiewicz. They are an important class of manifolds and have been extensively studied since their introduction. We will start with the definition and examples of quasi-toric manifolds. Then we will study their cohomology ring, smooth structure, and operations like connected sum and blow down.

The question of existence of symplectic structure and relation to toric varieties will be discussed. Stable almost complex structure will be introduced, leading to Chern classes. Existence of almost complex structure will also be studied. If time permits, quasi-toric orbifolds will also be discussed. We will mention several open problems and possible directions of research.

References

1. V. M. Buchstaber and T. E. Panov, Torus actions and their applications in topology and combinatorics, University Lecture Series **24**, American Mathematical Society, Providence, RI, 2002.
2. V. M. Buchstaber and N. Ray, Tangential structures on toric manifolds, and connected sums of polytopes, Internat. Math. Res. Notices **2001**, no. 4, 193–219.
3. M. W. Davis and T. Januszkiewicz, Convex polytopes, Coxeter orbifolds and torus actions, Duke Math. J. **62** (1991), no.2, 417–451.
4. M. Poddar and S. Sarkar: On quasitoric orbifolds, Mathematics arXiv:0809.3132
5. S. Ganguli and M. Poddar: J-holomorphic geometry of four dimensional quasitoric spaces, in preparation.

3. *Structures on manifolds* Having additional 'structures' on a smooth manifold often results interesting topological consequences. The focus on these lectures will be understanding such consequences in the context of the following structures: (i) (almost) complex structures, (ii) Kähler structure and (iii) symplectic structures. Illustrative examples of each kind will be discussed. Study of relation between symplectic and Kähler structures is a major research theme.

References

1. J. W. Milnor and J. D. Stasheff, Characteristic classes, Annals of Math. Stud. **76**, Princeton Univ. Press, NJ.
2. F. Hirzebruch, Topological methods in algebraic geometry, Springer.

About five Guest Lectures and five participant-lectures are envisaged to be held during the Workshop.